Find the Norton equivalent of the network to the left of terminals ab in Fig. 1 if element X is: (a) a 5-Ω resistor; (b) a 3-A independent current source, arrow directed upward; (c) a dependent voltage source, of magnitude 2I_1, positive reference at top.
Solution: Since the problem is to find the Norton equivalent of the network to the left of terminals ab, all the circuitry to the right is not important.

Fig. 3 (a) \[ \begin{array}{c}
20\Omega \\
5\Omega \\
b
\end{array} \quad \Rightarrow \quad \begin{array}{c}
10\Omega \\
a \\
4\Omega \\
\Rightarrow \\
b \\
\end{array} \quad \begin{array}{c}
14\Omega \\
\end{array} \quad (b) \quad \begin{array}{c}
a \\
b
\end{array}
\]

With element X a 5-Ω resistor, \( i_{sc} = 0 \) and \( R_n \) is found

\[ R_n = 14\Omega. \]

(b) Fig. 4 shows the circuit for which the Norton equivalent must be determined.

Fig. 4

If we short terminals ab we can write a KCL equation and solve for \( i_{sc} \). Fig. 5 shows the circuit.

\[ 3 = v \left( \frac{1}{20} + \frac{1}{10} \right) \]

\[ v = \frac{3}{0.15} = 20V \]

\[ i_{sc} = \frac{20V}{10\Omega} = 2A. \]

Replacing the 3-A source with an open circuit gives \( R_n \).

Fig. 6

\[ R_n = 10 + 10 + 10 = 30\Omega \]

Fig. 7

Fig. 7 shows the resulting Norton equivalent circuit.

108
(c) Fig. 8 shows the new circuit.

\[ i_1 \]

Fig. 8

Connect a 1-A current source across the terminals ab as shown in Fig. 9.

By writing a KVL equation for the loop on the right

\[ V = I \cdot R \]

is found.

\[ 2i_1 = i_1 \cdot (10) + v \]

Since \( i_1 = -1 \)

We have

\[ 2 ( -1 ) = -1 \cdot (10) + v \]

\[ v = 8V \]

\[ R_n = \frac{V}{I} = 8\Omega. \]

With no independent sources to the left of terminals ab \( i_{sc} = 0 \).

**PROBLEM 1-13**

In the circuit shown in Fig. 1, find \( v_1 \) and the voltage \( v \) across both sources if \( i_s \) is given as 12A.

![Fig. 1](image)

![Fig. 2](image)
Solution: In Fig. 2 observe that
\[ i = i_s + 0.6v_1 = i_1 + i_2. \]

Since \( i_1 \) and \( i_2 \) both flow through a branch with the same equivalent resistances, they must be equal. Thus we have
\[ i = i_s + 0.6v_1 = 2i_1 = 2i_2. \]

We can write \( v_1 = i_1R \) where \( R = 2 \Omega \). Since \( i = 2i_1 \) we have
\[ i_1 = \frac{1}{2}i = \frac{1}{2}(i_s + 0.6v_1) \]
\[ v_1 = i_1R = \frac{1}{2}(i_s + 0.6v_1)R \]

Substituting 2-\( \Omega \) of \( R \) and 12A for \( i_s \), solve for \( v_1 \)
\[ v_1 = \frac{1}{2}(12 + 0.6v_1) \]
\[ v_1 = 12 + 0.6v_1 \]
\[ 0.4v = 12 \]
giving \[ v_1 = \frac{12}{0.4} = 30V. \]

\( v \) is found by calculating \( i \) and the equivalent resistance \( R_{eq} \) across the terminals of the two sources.

From Fig. 3 we find \( v = R_{eq} i = 7.5(30) = 225V. \)

![Fig. 3](image-url)
The circuit shown in Fig. 1 is a linear equivalent circuit of a transistor circuit. The source \( v \) is an ideal independent voltage source, but \( h_{11}i_1 \) and \( h_{12}v_2 \) are dependent current and voltage sources, respectively. Determine Thevenin's equivalent circuit with respect to terminals \( a - b \).

**Fig. 1**

**Fig. 2**

Solution: The first step in finding the Thevenin equivalent of a circuit is to find the open circuit voltage, \( (v_2)_{OC} \) which replaces all the sources in the Thevenin equivalent circuit.

In the right half of the circuit of Fig. 1 note that

\[
(v_2)_{OC} = r_2 h_{21} i_1
\]

where \( i_1 \) is found from the left half of the circuit

\[
i_1 = \frac{v - h_{12} v_2}{r_1}
\]

113
Hence, substituting Eq.(2) into (1) yields

$$(v_2)_{oc} = r_2 \frac{h_{21}}{h_{12}} \left[ \frac{V - h_{12}v_2}{r_1} \right]$$  \hspace{1cm} (3)

But since $v_2 = (v_2)_{oc}$, solve for $(v_2)_{oc}$,

$$(v_2)_{oc} = r_2 \frac{h_{21}}{r_1} \left[ \frac{V - h_{12}(v_2)_{oc}}{r_1} \right]$$

$$(v_2)_{oc} = r_2 \frac{h_{21}}{r_1} \frac{V - h_{12}(v_2)_{oc}}{r_1} - r_2 \frac{h_{21}}{r_1} \frac{h_{12} (v_2)_{oc}}{r_1}$$

$$(v_2)_{oc} \left[ 1 + r_2 \frac{h_{21}}{r_1} \frac{h_{12}}{r_1} \right] = r_2 \frac{h_{21}}{r_1} \frac{V}{r_1}$$

$$(v_2)_{oc} = \frac{r_2 \frac{h_{21}}{r_1} \frac{V}{r_1}}{1 + r_2 \frac{h_{21}}{r_1} \frac{h_{12}}{r_1}}$$

obtaining,

$$(v_2)_{oc} = \frac{r_2 \frac{h_{21}}{r_1} \frac{V}{r_1}}{r_1 + r_2 \frac{h_{21}}{r_1} \frac{h_{12}}{r_1}} = \frac{r_2 \frac{h_{21}}{r_1} \frac{V}{r_1}}{r_1 + r_2 \frac{h_{21}}{r_1} \frac{h_{12}}{r_1}}$$

In order to find the Thevenin impedance $Z_s$, short circuit terminals $a - b$ and find the current $i_s$ flowing from $a$ to $b$ which yields

$$Z_s = \frac{(v_2)_{oc}}{i_s}$$

$i_s$ is to be the current produced by the dependent current source $h_{21} i_1$.

Since it has already been established that

$$i_1 = \frac{V - h_{12} v_2}{r_1}$$

$i_1$ must be $\frac{V}{r_1}$ when the terminals $a - b$ are short-circuited ($v_2 \equiv 0$). Hence,

$$i_s = \frac{h_{21} v}{r_1}$$ and

$$Z_s = \frac{(v_2)_{oc}}{i_s} = \frac{r_1 \frac{r_2}{r_2 + r_1}}{r_1 + r_2 \frac{h_{21}}{r_1} \frac{h_{12}}{r_1}}$$

Since $h_{21}$ and $h_{12}$ are real positive numbers $Z_s$ is a pure resistance, $R_s$.

The Thevenin equivalent circuit is the Thevenin source $(v_2)_{oc}$ in series with the Thevenin resistance $R_s$ shown in Fig. 2.

$R_s$ is called the output impedance and the dimensionless quantity $\frac{h_{21} \frac{r_2}{h_{12} h_{21}}}{r_2 + r_1}$ is called the voltage gain.
Use superposition to find $v$ in each of the circuits shown in Fig. 1.

![Circuit Diagram](image)

Fig. 1 (a) (b) (c)

Solution: (a) Replace the 5-V source on the right with a short circuit and find the currents through the simplified circuit in Fig. 2.

![Circuit Diagram](image)

Fig. 2 Fig. 3 Fig. 4

Note that $i = i_1 + i_2$ and, since both $i_1$ and $i_2$ flow through a 4-Ω resistor,

$$i_1 = i_2 \quad \text{and} \quad i = \frac{i_2}{2} = \frac{i_1}{2}.$$
Combining the two 4-Ω resistors yields the circuit in Fig. 3.

\[ i = \frac{6}{4 + 2} = 1 \text{ A} \]

\[ i_1 = \frac{1}{2} \text{ A}, \quad v_a = (\frac{1}{2}) \times 4 = 2 \text{ V}. \]

Go back to the original circuit in Fig. 1(a) and replace the 6-V source on the left with a short circuit, and solve for the currents in the simplified circuit shown in Fig. 4.

From the circuit in Fig. 4, the circuit in Fig. 5 can be derived. Solving for \( i_3 \), we have

\[ i_3 = \frac{6}{4 + 2} = 1 \text{ A}; \quad v_b = (1)(4) = 4 \text{ V}. \]

By adding the currents, obtained by analyzing the effect of one source at a time on the circuit the current, when all sources are connected, can be found.

Thus,

\[ (i_1 + i_3) \times 4 = v \]

\[ \left( \frac{1}{2} + 1 \right) \times 4 = 6 \text{ V}. \]

(b) Applying the theorem of superposition yields the two circuits in Figs. 6 and 7.

![Fig. 6](image)

![Fig. 7](image)

In the circuit of Fig. 6, apply KCL to the voltage at node \( n_1 \).

\[ 6 = v_{n_1} \left( \frac{1}{.5} + \frac{1}{.4} + \frac{1}{.6} \right) \]

\[ v_{n_1} = \frac{6}{\frac{1}{.5} + \frac{1}{.4} + \frac{1}{.6}} = \frac{6}{1.111 + 1.667} = 2.16 \text{ V}. \]
The voltage $v_a$, due to 6-A source alone is
\[ v_a = i(0.5) = \left( \frac{v_{n1}}{.5 + .4} \right) (0.5) \]
\[ v_a = \frac{2.16}{.9} (0.5) = 1.2V. \]

In the circuit of Fig. 7, apply KCL to find the voltage at node $n_2$
\[ - (-21) = v_{n2} \left( \frac{1}{.5 + .6} + \frac{1}{.4} \right) \]
\[ v_{n2} = \frac{21}{.5 + .6} + \frac{1}{.4} = 21 \frac{.909 + 2.5}{2} = 6.16 \text{ V}. \]

The voltage $v_b$, due to the -21 A source alone is
\[ v_b = i(0.5) = \left( \frac{-v_{n1}}{.5 + .6} \right)(0.5) \]
\[ v_b = \left( \frac{-6.16}{1.1} \right) (0.5) = -2.8 \text{ V}. \]

Obtain $v$ by adding the two voltages
\[ v = v_a + v_b = 1.2 - 2.8 = -1.6 \text{ V}. \]

(c) Applying the superposition theorem, obtain three circuits from Fig. 1(c) shown in Figs. 8 - 10.

180 mA and 120 mA sources open circuited

16V source short circuited and 120 mA source open circuited

Fig. 8

Fig. 9
Superposition gives $v = v_a + v_b + v_c$

Note that conductance values are given.

Writing the KVL equation around the loop in Fig. 8 yields

$$16 = i_a \left( \frac{1}{2} + \frac{1}{6} + \frac{1}{3} \right) \times \frac{1}{10^{-3}}$$

$i_a = 16 \text{ mA}$

$v_a = -\frac{16 \text{ mA}}{2 \text{ m}} = -8 \text{ V}.$

Writing the KCL equation for node $n_1$ in Fig. 9 yields

$$180 \text{ mA} = v_{n_1} \left( \frac{2 \cdot 3}{2 + 3} + 6 \right) \times 10^{-3}$$

$$v_{n_1} = \frac{180}{\left( \frac{6}{5} + 6 \right)} = 25 \text{ V}$$

$$v_b = -\frac{i_b}{2m} = -\frac{v_{n_1} \left( \frac{6}{5} \right)}{2} = -15 \text{ V}.$$  

Writing the KCL equation for node $n_2$ in Fig. 10 gives

$$120 \text{ mA} = v_{n_2} \left( \frac{2 \cdot 6}{2 + 6} + 3 \right) \times 10^{-3}$$

$$v_{n_2} = \frac{120}{\frac{12}{8} + 3} = 26.667 \text{ V}$$

$$v_c = \frac{v_{n_2} \left( \frac{12}{8} \right)}{2} = 20 \text{ V}.$$  

Applying superposition

$$v = v_a + v_b + v_c$$

$$v = (-8) + (-15) + (20) = -3 \text{ V}.$$