1: Let $S$ be a set of real numbers which is closed under multiplication (that is, if $a$ and $b$ are in $S$, then so is $ab$). Let $T$ and $U$ be disjoint subsets of $S$ whose union is $S$. Given that the product of any three (not necessarily distinct) elements of $T$ is in $T$ and that the product of any three elements of $U$ is in $U$, show that at least one of the two subsets $T, U$ is closed under multiplication.

2: Let $S$ be the set of ordered triples $(a, b, c)$ of distinct elements of a finite set $A$. Suppose that

1. $(a, b, c) \in S$ if and only if $(b, c, a) \in S$.
2. $(a, b, c) \in S$ if and only if $(c, b, a) \notin S$.
3. $(a, b, c)$ and $(c, d, a)$ are both in $S$ if and only if $(b, c, d)$ and $(d, a, b)$ are both in $S$.

Prove that there exists a one-to-one function $g$ from $A$ to $\mathbb{R}$ such that

$$g(a) < g(b) < g(c)$$

implies $(a, b, c) \in S$. Note: $\mathbb{R}$ is the set of real numbers.